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Structural Sizing Considerations for Large Space Platforms

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Structural optimization studies are made using mathematical programming techniques to examine minimum mass structural proportions of deployable and erectable tetrahedral truss platforms subject to the integrated effects of various design requirements. Considerations integrated into the optimization process are: 1) lowest natural frequencies of the platform and individual platform components (struts); 2) packaging constraints imposed by the Shuttle cargo bay capacity; 3) initial curvature of the struts; 4) column buckling of the struts due to gravity gradient, orbital transfer, strut length tolerance, or design loads; and 5) lower limits for strut diameter and wall thickness. Extremely low-mass designs are shown to be theoretically possible with strut proportions much more slender than those conventionally used for Earth bound application. The practical limits of slender strut construction, however, are not established.

Nomenclature

A	= planform area of truss platform, Eq. (A1)
A_{cb}	= cross-sectional area of Shuttle cargo bay
A_f, A_c	= cross-sectional areas of face and core struts, respectively
A_p	= cross-sectional area of packaged deployable and erectable platforms, Eqs. (4) and (9)
D	= maximum dimension of hexagonal truss platform
D_T	= platform bending stiffness, Eq. (A6)
d_c, d_f	= diameters of core and face struts, respectively
d_{cb}	= diameter of Shuttle cargo bay
d_p	= maximum dimension of hexagonal cross-sectional area of packaged deployable truss platform, Eq. (3)
d_1, d_2	= minimum and maximum diameter, respectively, of tapered nestable strut
E	= modulus of elasticity
f_d	= truss platform design fundamental frequency
f_T	= truss platform fundamental frequency, Eq. (A7)
f_s	= strut fundamental frequency, Eqs. (A10) and (A12)
g_ξ	= constant defined by Newton's second law, Eq. (A8)
g_0	= Earth's gravitational acceleration (9.81 m/s^2)
H	= truss platform depth, Eq. (A2)
l_c, l_f	= lengths of core and face struts, respectively
l_{cb}	= length of Shuttle cargo bay
M	= structural mass (struts + joints)
M_j	= total mass of all joints
M_{sys}	= total system mass (struts + joints + payload)
M_0	= total startburn mass
m_p	= distributed payload mass supported by truss platform
N_c, N_f	= number of nestable strut halves per stack for core and face struts, respectively, Eq. (8)
P_d	= strut design load
P_E	= Euler buckling load of strut
P_{gg}	= strut load due to gravity gradient considerations
P_{ot}	= strut load due to orbital transfer considerations
R	= orbit radius (6740 km for LEO)
r_{cl}	= cluster (nodal) joint radius, Eq. (1)
R_e	= Earth radius (6373 km)
S	= center-to-center spacing of cluster joints in deployable platform packaged configuration
SF	= number of Shuttle flights required for transportation to low Earth orbit
t_c, t_f	= strut wall thicknesses of core and face struts, respectively
T	= thrust
W_0	= startburn weight
Δ	= nestable strut stacking increment, Eq. (7)
δ	= lateral center deflection of initially curved strut
λ	= minimum-to-maximum diameter ratio for nestable strut

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Introduction

THE availability of Space Shuttle for transporting large payloads to orbit has given rise to prospective missions involving large truss structures ranging from 100-m reflector (antenna) platforms to huge (kilometer size) solar collector platforms. These missions involve structural sizes and operational requirements for which little information exists concerning efficient structural proportions—a feature which can impact Shuttle transportation efficiency.

Previous studies of ways to accomplish various missions with Shuttle include investigations of structures which are deployable (unfolded on-orbit[1,2]), erectable (assembled on-orbit[3-8]), and space fabricated (manufactured and assembled on-orbit[8-10]). These investigations may be generally classified as either design studies for specific size structures and element concepts, or system level studies for complex projects requiring huge, built-up structural elements.

It is the purpose of this investigation to apply a preliminary automated structural sizing procedure for structural platforms to: 1) provide baseline designs for comparison of minimum mass platforms which are configured for Shuttle transport to orbit as either deployable or erectable structures, 2) identify the efficient range of application of each concept, and 3) identify and quantify key parameters to which mass and transportation requirements of efficiently proportioned structures are sensitive. It should be noted that many of the optimized designs displayed herein have extremely slender struts which greatly exceed conventional limits for Earth based structures. The practical limit of slender strut construction methods, however, are not established.

Platform Configuration and Packaging

Because of its high stiffness and low mass[2] a tetrahedral truss structure with a hexagonal planform is selected for study (Fig. 1a). The struts making up the surfaces of the platform (face struts) may have different proportions from the in-

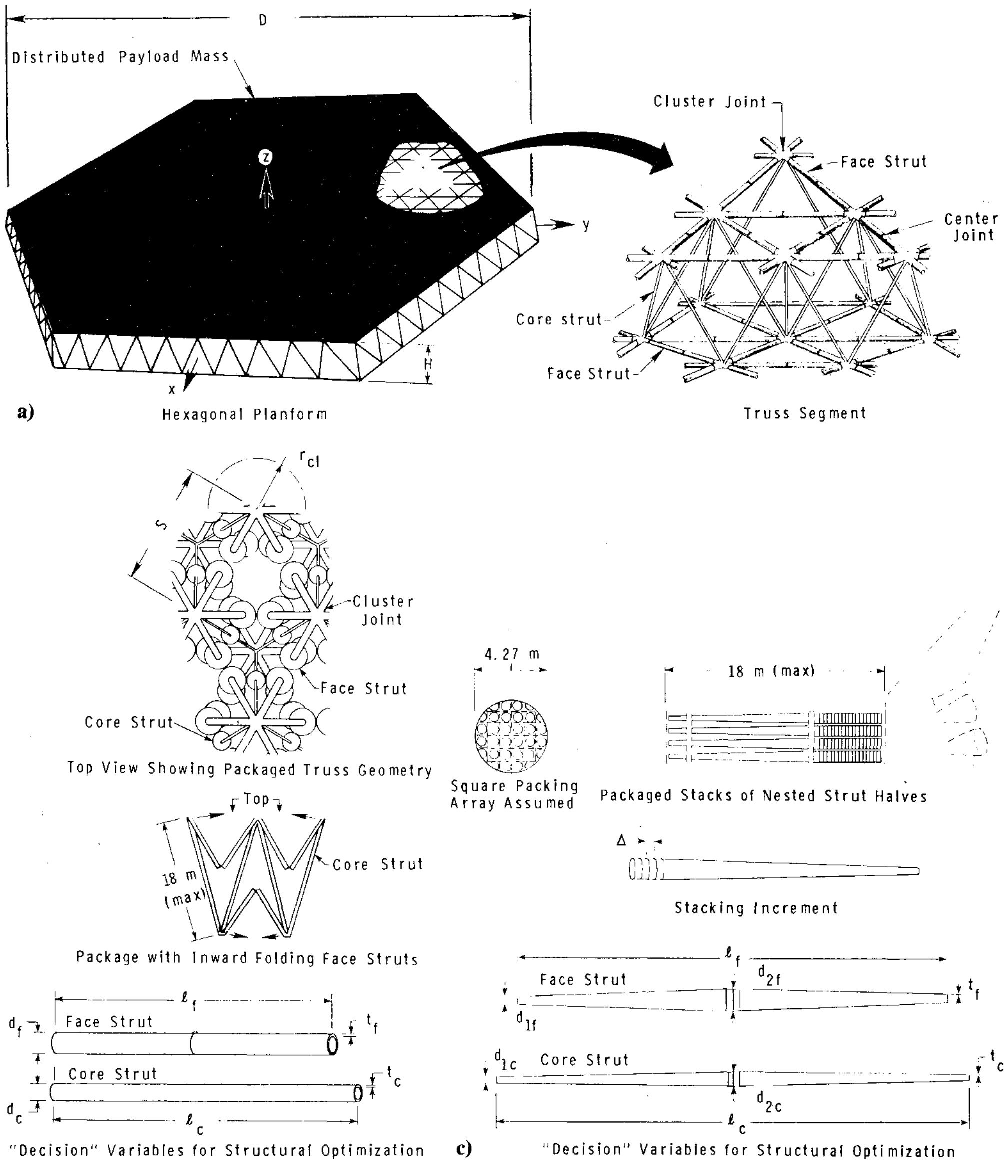

Fig. 1 Tetrahedral truss nomenclature and packaging geometry: a) tetrahedral truss platform, b) deployable packaging, and c) errectable packaging.

tersurface or "core" struts of the platform. Joint masses are proportional to strut diameter with mass factors taken from proven laboratory specimens believed to be representative of flight-weight hardware. A nonstructural (payload) mass is assumed to be distributed over the platform. Two types of platforms are considered: deployable and erectable. The deployable platform is unfolded on-orbit, while the erectable platform must be assembled on-orbit. Each type of construction imposes unique constraints on the platform design through the way each concept packages for Shuttle transportation to orbit.

Deployable Platform Packaging

The deployable platform is assumed to be constructed of cylindrical struts. Figure 1b shows the packaging arrangement used for the present studies. The surface struts are hinged at their centers to fold inward while the core struts are one piece. The platform hinged in this manner folds into a package with a hexagonal cross section. The maximum allowable length of the package is 18 m, slightly less than the length of the Shuttle cargo bay. With inward folding, the face struts cannot be longer than the core struts. The area and volume requirements of this package are functions of the strut diameter d, thickness t and length l. The radius of the cluster joint (Fig. 1b), required for parallel folding (and compact packaging) of the struts is given by:

$$r_{cl} = \tfrac{1}{4}\{d_f + [3(3d_f+2d_c)(d_f+2d_c)]^{1/2}\} \tag{1}$$

where the subscripts f and c throughout this paper refer to the face and core struts, respectively. The center-to-center spacing, (Fig. 1b), of the cluster joint in the packaged configuration is given by:

$$S = 2r_{cl} + d_f \tag{2}$$

The maximum dimension of the package cross section and the package cross-sectional area, respectively, are:

$$d_p = SD/l_f \tag{3}$$

$$A_p = (3\sqrt{3}/8)d_p^2 \tag{4}$$

The maximum allowable value of d_p per Shuttle flight is taken to be 4.27 m, which is slightly less than the diameter of the Shuttle cargo bay. Hence, on a cross-sectional area basis, it is assumed that the number of Shuttle flights required to transport a given platform can be approximated as:

$$SF = \frac{A_p}{A_{cb}} = \frac{3\sqrt{3}}{2\pi}\left(\frac{d_p}{d_{cb}}\right)^2 \tag{5}$$

If the package is mass critical, the number of Shuttle flights required is simply the total system mass of the platform divided by the Shuttle lift capability:

$$SF = M_{sys}/29{,}480\ \mathrm{kg} \tag{6}$$

where A_{cb} and d_{cb} are the area and diameter of the Shuttle cargo bay, respectively. The problem of joining segments of a deployable structure when multiple Shuttle flights are required is not addressed herein.

Erectable Platform Packaging

The erectable platform truss is constructed of tapered, nestable struts[3] which are packaged in the Shuttle in stacks of strut halves, as shown in Fig. 1c. The stacks may not exceed 18 m in length. Packaging equations[3] are modified in the present paper to include unequal face and core struts. The stacking increment, Δ (see Fig. 1c), is given in terms of the average diameter $\bar{d}$ of the strut and the ratio of the minimum-to-maximum strut diameters, λ:

$$\Delta = \frac{t}{\bar{d}}\frac{1+\lambda}{1-\lambda}\left[d^2\left(\frac{1-\lambda}{1+\lambda}\right)^2 + \left(\frac{l}{2}\right)^2\right]^{1/2} \tag{7}$$

The number of strut halves per stack is

$$N = [(l_{cb} - \tfrac{1}{2}l)/\Delta] + 1 \tag{8}$$

It is understood that the variables in Eqs. (7) and (8) are to be subscripted with f or c to denote either face or core strut values, respectively. A square packing array[3] is assumed for the cross-sectional packaging arrangement of the stacks so that the maximum diameters, d_2, of the face and core struts determine the cross-sectional area, A_p, required for stowage:

$$A_p = \frac{2n_f d_{2f}^2}{N_f} + \frac{2n_c d_{2c}^2}{N_c} \tag{9}$$

where n_f and n_c are the total number of face and core struts, respectively, [Eqs. (A3) and (A4)]. Dividing Eq. (9) by the cross-sectional area of the Shuttle cargo bay approximates the number of Shuttle flights required on an area basis:

$$SF = A_p/\pi(d_{cb}/2)^2 \tag{10}$$

When the strut dimensions result in mass critical payloads, the required number of Shuttle flights is given by Eq. (6).

Analytical Methods

Optimization Procedure

To arrive at an optimum structural design, it is first necessary to define a function describing a characteristic of the structure which can be advantageously optimized with respect to a set of structural design variables. This function is termed the "objective function." The values of the design variables (termed "sizing variables") which make the objective function a minimum are taken as the optimum structural proportions. The optimum design may be constrained to meet any number of predetermined operational requirements and limiting dimensional considerations. The optimization procedure herein employs the CONMIN Computer Program[11] for the solution of linear and nonlinear constrained optimization problems.

Objective Function

The structural mass per unit area of the platform is taken as the objective function. This function may be expressed in terms of the mass of the struts plus the mass of the joints as:

$$M/A = (2\sqrt{3}/f^2)(2\rho_f A_f l_f + \rho_c A_c l_c) + M_j/A \tag{11}$$

Table 1 Sizing variables

Sizing variable	Face struts	Core struts	Limits
Wall thickness	t_f	t_c	≥ 0.508 mm
Diameter	d_f, d_{1f}, d_{2f}	d_c, d_{1c}, d_{2c}	$\geq = 1.27$ cm
Length	l_f	l_c	$\leq$ 18 m (Deploy), 36 m (Erect)

Table 2 Structural response

Structural response	Constraint
f_T, truss fundamental frequency (free edges)	$f_T \geq f_d$
f_s, strut fundamental frequency (simply supported)	$f_s \geq kf_d$
P, strut load (simply supported)	$P \leq P_E$

The joint masses for the cluster joint and strut center and end joints are scaled from laboratory specimens.[7] (Joint mass ranged between 15 and 30% of strut mass.)

Sizing Variable

An examination of Eq. (11) shows that M/A is a function of the structural proportions (sizing variables) of the struts. The sizing variables (Fig. 1) and their limits used in the present paper are shown in Table 1.

Constraints and Other Structural Considerations

Simple analytical relations were developed for the platform structural response. These relations are similar to those for rectangular planform, regular tetrahedral platforms constructed of cylindrical struts.[5] The equations used in the present study (see Appendix) are used to specify structural response standards which the design may be constrained to satisfy. The structural response relations and corresponding constraints used for the present paper are shown in Table 2 where f_d is the platform design frequency (specified); and P is 1) P_d, assembly, docking, maneuvering, load (specified); 2) P_{gg} [gravity gradient load, Eq. (16)]; 3) P_{ot} [orbital transfer load, Eq. (A18)]; 4) P_l [strut length error load, Eq. (A9)]. Other effects which impact the structural design are also considered. Such effects include curvature of the strut axis, Eq. (A11), and strut taper, Eq. (A14). Both of these effects reduce strut axial stiffness and, ultimately, platform bending stiffness.

Results and Discussion

Calculations are performed to provide structural mass per unit area, Shuttle transportation, and structural proportion requirements for both deployable and erectable platforms. The sensitivity of these efficient designs to other parameters such as: 1) distributed payload mass, 2) strut-to-platform design frequency ratio, 3) strut initial curvature, 4) strut design load, and 5) orbital transfer load, is also examined. Only graphite-epoxy struts are considered (see Table 3).

Baseline Deployable Platforms

The structural mass per unit area and Shuttle transportation requirements to low Earth orbit for platforms with spans of 300 to 1000 m are shown in Fig. 2 as a function of platform design fundamental frequency. These baseline calculations were performed assuming perfectly straight struts, a strut frequency factor, $f_s/f_d = 10$ to insure that coupling does not occur between platform and strut modes, and a distributed payload mass of 0.1 kg/m², which is

Table 3 Graphite-epoxy material properties

	E_x, GPa	E_y, GPa	G_{xy},GPa	υ_{xy}	α_x, K^{-1}	α_y, K^{-1}
Undirectional	131	10.9	6.4	0.32	-0.54×10^{-6}	29×10^{-6}
Laminate $[90_{0.6}/0_{0.88}/90_{0.6}]$	117	25.4	6.4	0.138	0.22×10^{-6}	11×10^{-6}

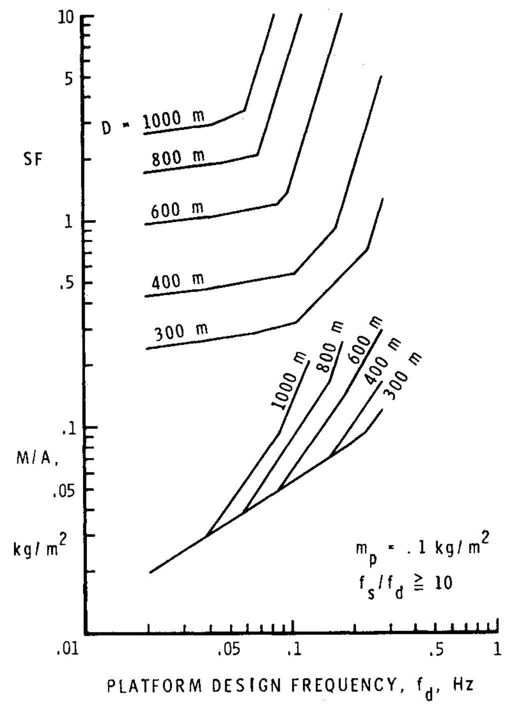

Fig. 2 Effect of deployable platform design frequency on structural mass per unit area and transportation requirements.

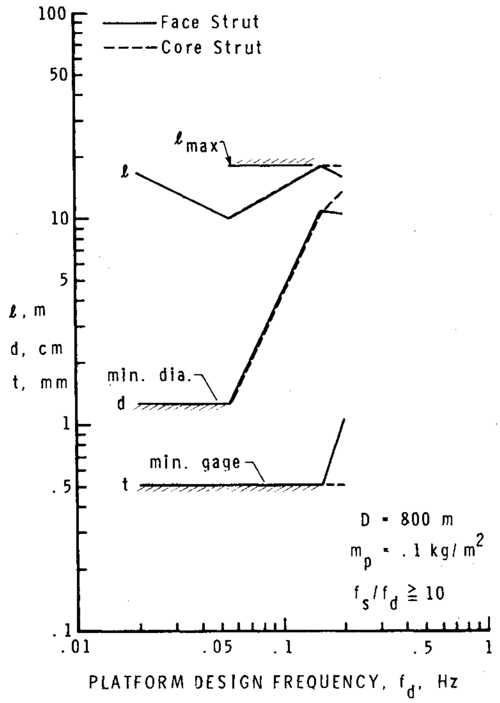

Fig. 3 Effect of 800-m deployable platform design frequency on strut geometry.

representative of a low mass reflector surface. The only load considered for these calculations was the control load required to overcome gravity gradient effects (found to be insignificant for these calculations).

The lower family of curves in Fig. 2 showing structural mass per unit area emanate from one limiting curve at various design frequencies. For the limiting curve the face and core strut thicknesses and diameters are at lower bounds and the strut lengths are constrained by $f_s/f_d=10$.

The Shuttle transportation requirements for these configurations are shown by the upper family of curves in Fig. 2. The horizontal portions of these curves are regions where Shuttle payloads are mass, rather than volume, limited. At higher values of platform frequency, the strut dimensions are sufficiently large to force platform package size to dominate the Shuttle payload and the slope of the curves becomes nearly vertical.

All curves in Fig. 2 show other milder slope changes which can be explained by a study of strut dimensions as functions of platform design frequency. Figure 3 shows this typical behavior for the 800-m platform. Structural proportions which characterize minimum mass designs are important. Strut slenderness ratios (length-to-radius of gyration) calculated from the data in Fig. 3 vary from approximately 500 at the higher platform frequencies to approximately 4000 at the lower platform frequencies. Conventional slender tubes are usually limited to slenderness values of less than 200.[1] Thus, fabrication of structural platforms using struts with nonconventional slenderness ratios may require advanced manufacturing techniques to insure strut straightness and length control.

Baseline Erectable Platforms

Figure 4 shows the structural mass per unit area and Shuttle transportation requirements for the erectable platform constructed of nestable struts.[3] These calculations are performed for conditions analogous to the deployable platform results shown in Fig. 2 except that face and core strut lengths are constrained to 36 m or less. The erectable struts have an additional parameter λ, the minimum-to-maximum diameter ratio which is constrained to a value of 0.5 or less because the packaging efficiency of nestable struts deteriorates [Eq. (7)] as λ approaches unity. It was found that for values of $\lambda=0.5$, the maximum mass penalty was approximately 2% and packaging efficiency was sufficient to insure mass limited Shuttle payloads. For the designs studied herein, the value of λ for the face struts was always 0.5, although for the core struts, λ ranged between 0.43 and 0.5.

The results for the erectable platform in Fig. 4 show that structural mass per unit area is approximately proportional to platform design frequency. However, the transportation requirements shown in Fig. 4 are all mass limited due to the high packaging efficiency of nestable struts. Figure 5 shows the strut structural proportions as a function of platform design frequency for the minimum mass 800-m erectable platform. From Fig. 5 the average slenderness ratios are found to vary from approximately 200 to 2000.

Deployable and Erectable Comparison

To illustrate the regions of application of deployable and erectable structures, the structural mass per unit area and transportation requirements of the baseline 400- and 800-m platforms are compared in Fig. 6. The mass per unit area results show that deployable and erectable platforms generally exhibit similar behavior over the frequency range investigated.

A more important result can be seen in the Shuttle transportation requirements by the upper family of curves in Fig. 6. This comparison shows that the number of Shuttle flights is

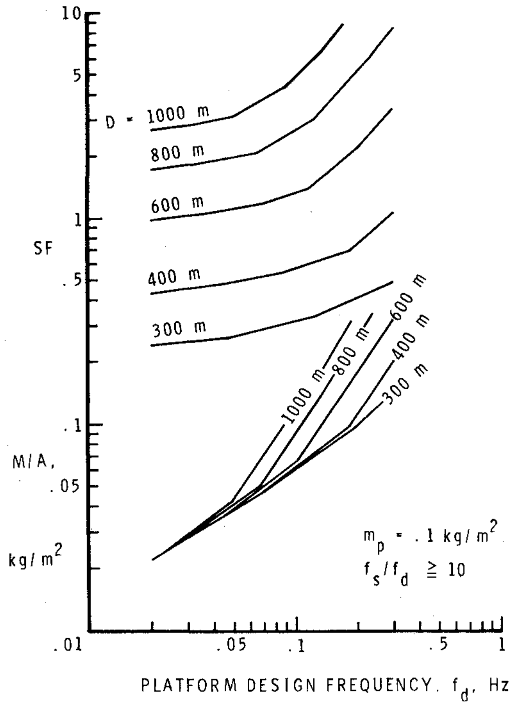

Fig. 4 Effect of erectable platform design frequency on structural mass per unit area and transportation requirements.

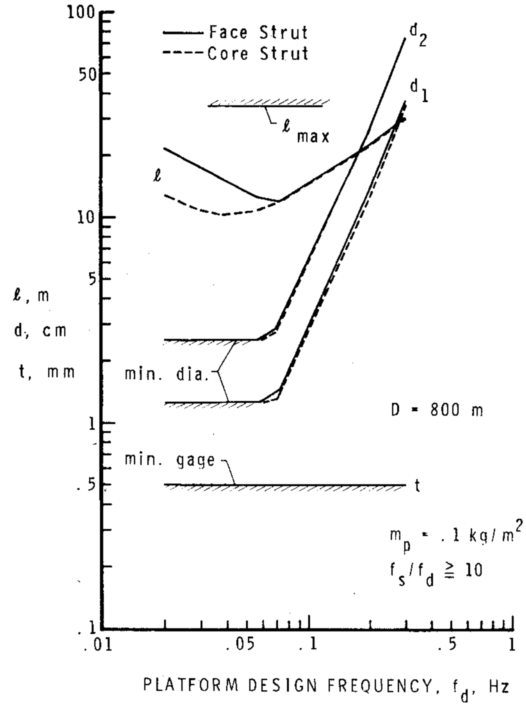

Fig. 5 Effect of 800-m erectable platform design frequency on strut geometry.

similar for deployable and erectable platforms designed for lower frequencies. However, at higher frequency requirements, Shuttle flights increase sharply for deployable platforms, while erectable platforms exhibit a more gradual increase. Thus, for a given size platform, nestable struts permit a stiffer structure than possible with deployable platforms without incurring transportation inefficiencies. Different design requirements will not alter this result; they will only change the frequency at which the deployable limit occurs.

Parametric Sensitivity Studies

Results presented thus far in Figs. 2-6 were made for selected values of certain parameters such as distributed payload mass m_p and strut frequency factor f_s/f_d. Initial strut curvature δ/l, strut design load P_d, and orbital transfer load P_{ol}, were taken to be zero in the baseline platform calculations. This section examines the sensitivity of structural mass per unit area and transportation requirements to variations in these parameters for both the 400- and 800-m platforms. Deployable and erectable platforms, with a design frequency, $f_d = 0.1$ Hz, are examined.

Distributed Payload

Baseline calculations were performed for an assumed membrane reflector-type distributed mass. Figure 7 shows the structural mass per unit area and Shuttle transportation requirements for variations in this parameter ranging from membrane reflector, $m_p \approx 0.1$, to solar collectors, $m_p \approx 1$, (cells) type surfaces. As can be seen, structural mass of the 400-m platform is not greatly affected for either deployable or erectable structure. Transportation requirements for both the 400-m deployable and erectable platforms are mass limited and increase nearly proportionally to the increase in distributed payload mass. The mass per unit area of the 800-m platforms is similar until the deployable platform depth becomes constrained by the core strut length limit of 18 m at $m_p \approx 0.3$ kg/m^2. For larger values of m_p the increased stiffness required to meet the specified design frequency cannot be achieved as efficiently with the deployable platform as with the erectable structure. The ratio of the structural mass to the total mass for all platforms considered is approximately 40-50% for a payload of 0.1 kg/m^2 but decreases to 8-15% for a payload of 2 kg/m^2.

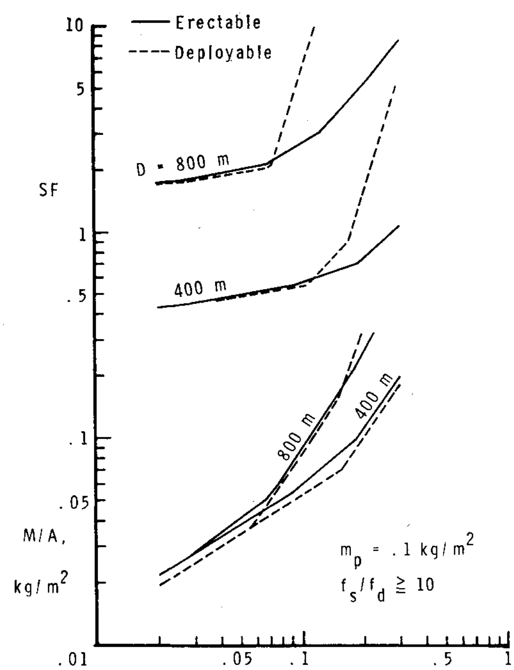

Fig. 6 Comparison of deployable and erectable platform structural mass per unit area and transportation requirements as a function of platform design frequency.

Strut Frequency

The effect of varying f_s/f_d is presented in Fig. 8. The mass per unit area requirements at the baseline strut frequency factor of 10 is approximately 4-5 times greater than at a factor of 2. The Shuttle flights required by the 400-m platforms are not adversely affected by the strut frequency factor.

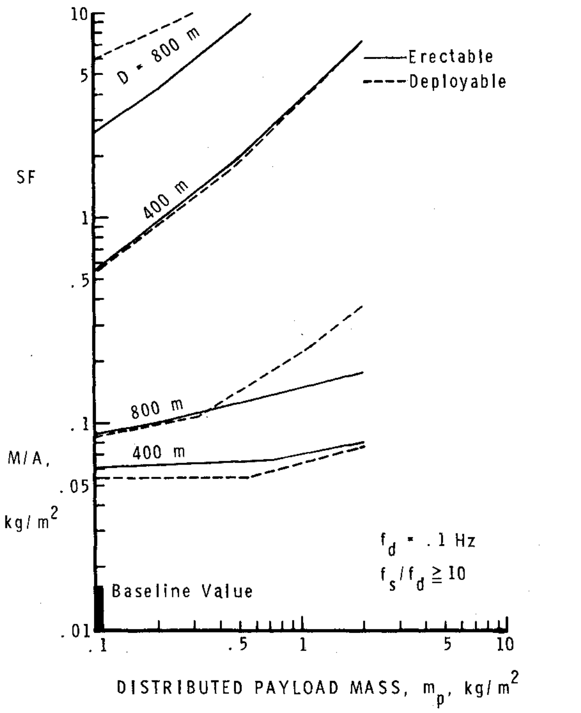

Fig. 7 Effect of distributed, nonstructural (payload) mass on platform structural mass per unit area and transportation requirements.

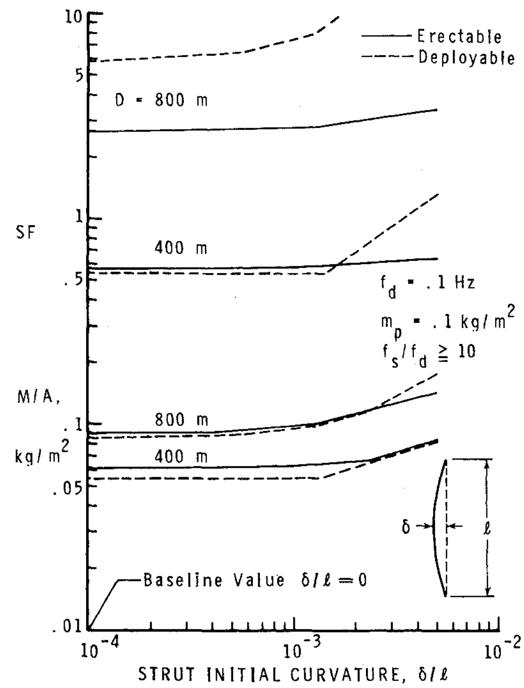

Fig. 9 Effect of strut initial curvature on platform structural mass per unit area and transportation requirements.

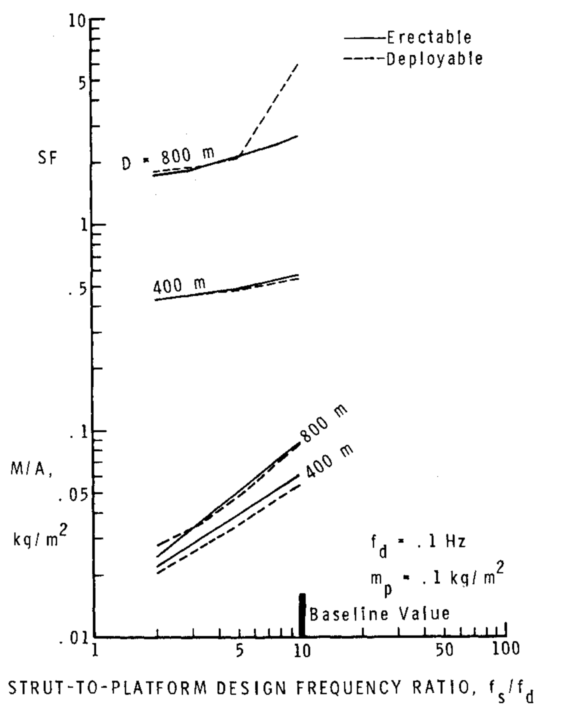

Fig. 8 Effect of strut-to-platform design frequency ratio on platform structural mass per unit area and transportation requirements.

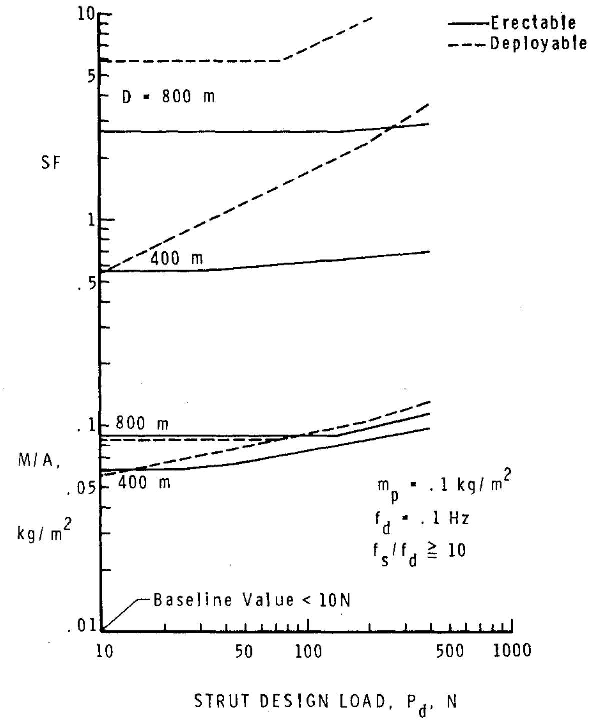

Fig. 10 Effect of strut design load on platform structural mass per unit area and transportation requirements.

However, an abrupt increase in Shuttle flights occurs for the 800-m deployable platform above a strut frequency factor of 5, indicating that a practical limit of this parameter probably exists for other large size deployable platforms.

Strut Initial Curvature

The effect of initial curvature is to reduce the axial stiffness of the face struts [Eq. (A11)] and thus reduce the bending stiffness of the platform. Consider the 800-m baseline deployable platform with perfect struts shown in Fig. 3. If a vibrational analysis is made for this design assuming a strut curvature δ/l of 0.001, the resulting frequency of the truss is decreased from 0.1 to 0.082 Hz. However, as shown in Fig. 9, sizing for this initial curvature will maintain the 0.1-Hz design frequency with only a small increase in structural mass. As strut curvature is increased, the platform becomes increasingly deeper to offset the loss in strut stiffness until the 18-m strut length constraint is reached. Beyond this point, both structural mass per unit area and Shuttle flights for the deployable platform increase abruptly.

For the 400-m deployable platform with $\delta/l = 0$, struts are minimum gage and the Shuttle flights are mass limited. In-

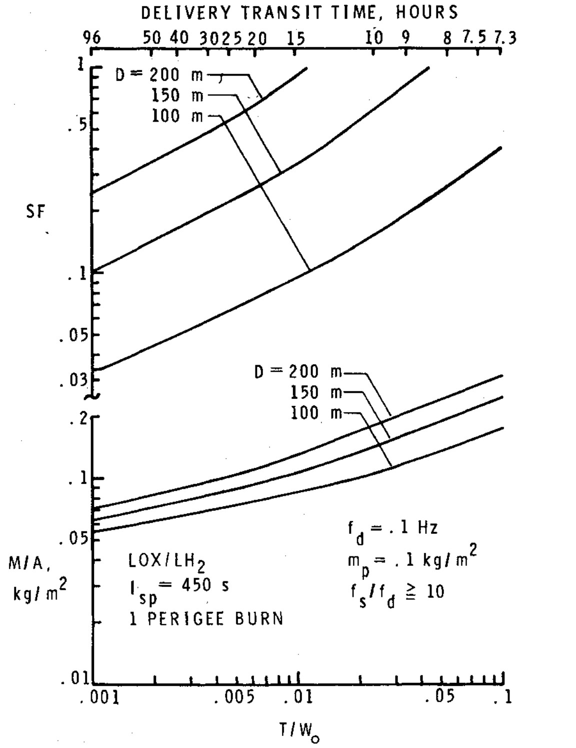

Fig. 11 Effect of thrust-to-weight ratio on structural mass per unit area and transportation requirements for a deployable platform that is transported from LEO to GEO.

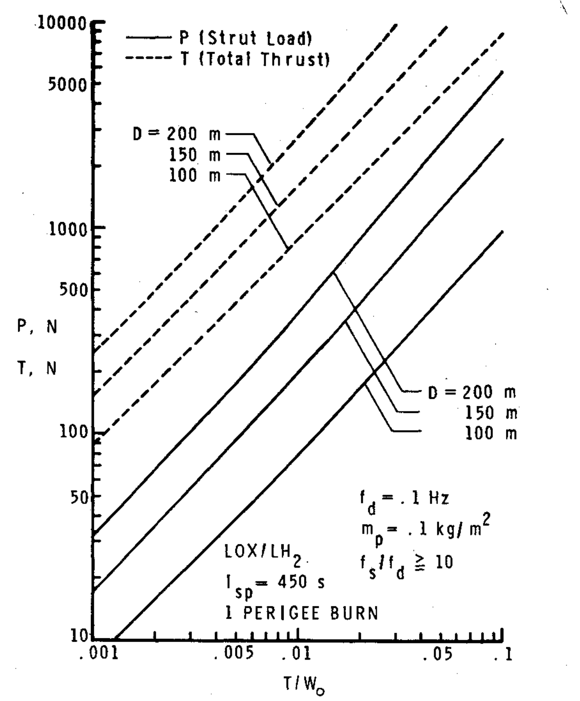

Fig. 12 Effect of thrust-to-weight ratio on total thrust required and on maximum core strut load for a deployable platform that is transported from LEO to GEO.

creasing strut curvature has little effect as long as minimum gage and minimum diameter struts are sufficient to meet design criteria. However, when strut dimensions exceed minimum limits sufficiently for Shuttle flights to become volume limited, transportation requirements can increase dramatically.

Strut Design Load

For the platforms studied herein, strut loads induced by gravity gradient control were insignificant. However, other loads such as docking, maneuvering, or assembly loads for erectable platforms could be significant. The effect of a constant strut design load is shown in Fig. 10. Shuttle transportation for the erectable platforms is relatively unaffected over the load range considered. The impact of strut design load on the Shuttle transportation for the 400-m deployable platform is significant, increasing from 0.5 flights for essentially zero design load, to approximately four flights for a design load of 400 N. The increased strut cross section required for the higher loads causes a packaging penalty which is reflected in the Shuttle transportation requirements for the 400-m deployable platform. The 800-m deployable platform Shuttle transportation requirements indicate that the larger strut cross sections required to satisfy frequency constraints are sufficient to carry strut loads up to approximately 100 N. Above this value, strut cross section increases significantly to carry the load, as shown by the increased Shuttle flight requirements.

Orbital Transfer Loads

An initial assessment of structural loads resulting from transferring minimum mass platforms from low Earth orbit (LEO) to geosynchronous orbit (GEO) is made. The effect of such loads on platform structural mass per unit area and Shuttle flight requirements is examined. These studies are limited to considering only constant thrust chemical propulsion systems and deployable platforms. The availability of a given thrust level engine is not considered. Instead, an estimate of the required thrust for various size platforms is made based on minimum mass structural sizing results. Using the orbital transfer vehicle sizing capability,[12,13] the propulsion system mass required to transfer a given spacecraft from LEO to GEO is determined for initial values of thrust-to-weight ratio, T/W_0, of 0.001 through 0.1 and a wide range of spacecraft masses.

The propulsion system thrust load is applied normal to the tetrahedral truss back-face at three nodal hard points located symmetrically about the center of gravity of the hexagonal planform truss. Equation (A18) gives the load induced in those core struts which transmit the applied thrust into the surrounding structure. Although strut loads decay away from the thrust application points, this decay is not considered herein and all struts are designed to carry the maximum load, given by Eq. (A18).

Using this approach, deployable platforms of 100-, 150-, and 200-m spans are sized for $T/W_0=0.001$, 0.01, and 0.1. Results for the minimized structural mass per unit area and Shuttle flight requirements of the platforms are shown in Fig. 11. (Shuttle transportation requirements for the propulsion system are not shown.) For the conditions specified, these results indicate the maximum size platform that could be placed in GEO, using one Shuttle flight to LEO, is approximately 200 m for $T/W_0=0.01$. If faster orbital transfer times are required, multiple Shuttle flights are required for a 200-m deployable platform.

The maximum strut loads which result from the orbital transfer maneuver are shown in Fig. 12 as a function of T/W_0 for three platform sizes. Strut loads are shown to increase rapidly with T/W_0. An estimate of the total constant thrust required to accomplish orbital transfer of the platforms is also shown in Fig. 12. Total thrust requirements vary from 100 to 300 N at $T/W_0=0.001$ and from 1000 to 3000 N at $T/W_0=0.01$.

Concluding Remarks

Minimum mass designs of large deployable and erectable tetrahedral truss platforms which meet a variety of constraints are identified using mathematical programming techniques. Ultra low-mass designs are shown to be possible for both deployable and erectable platforms. These designs exhibit values of structural mass per unit area which are characteristic of mesh reflector surfaces. Platform stiffness (frequency) requirements are shown to be a fundamental structural design driver for large space platforms, indicating a need to determine the minimum required value which will permit mission accomplishment. Strut proportions for minimum mass deployable and erectable platforms are found to be much more slender than struts conventionally used for Earth-bound application. If the advantages of such structures are to be realized, a fabrication capability for long, slender and straight, thin-walled composite tubes, both cylindrical and tapered, must be demonstrated.

For structures with sufficiently low design stiffness requirements, minimum mass deployable and erectable platforms require about the same number of Shuttle flights for transportation to orbit. However, for higher design stiffness requirements and/or more severe design constraints, the associated increase in strut diameter causes a large increase in Shuttle transportation requirements for deployable platforms and limits their range of applicability. Erectable platforms, which are not found to be limited in this manner due to the more efficient packaging capability of nestable struts, offer a possible alternative for platforms with stiffness requirements that cannot be efficiently met by a deployable structure.

Sensitivity studies show several key parameters which have significant impact on platform structural design. The severe effect on structural proportions of maintaining high strut frequency relative to the platform, indicates a need to determine the minimum required value for this parameter. The axial stiffness of the slender, minimum mass struts and thus the platform bending stiffness, is shown to be sensitive to initial imperfections in strut straightness. This indicates a need to include in the sizing process strut curvature resulting from any source such as manufacturing, gravity effects, or lateral acceleration due to maneuvering and orbital transfer. In general, strut design loads are found to have a more detrimental effect on deployable platforms than erectables, primarily due to the resultant increase in transportation requirements. Preliminary orbital transfer investigations indicate that deployable platforms of up to 200-m span require a single Shuttle flight to LEO and may be transferred to GEO using a constant thrust chemical propulsion system which limits initial acceleration to 0.01 *g* or less.

Appendix: Summary of Equations

The following equations are a summary of the structural design relations used in the optimization procedure. These equations are similar to those for regular, tetrahedral truss, rectangular platforms with cylindrical struts.[5] The present equations, however, are applicable to a hexagonal platform tetrahedral truss with cylindrical or tapered (nestable), dissimilar face and core struts.

General Truss Equations

The planform area of a hexagonal truss platform of maximum dimension D (see Fig. 1) is given by:

$$A=(3\sqrt{3}/8)D^2 \tag{A1}$$

The platform depth is given by:

$$H=l_f\left[\left(\frac{l_c}{l_f}\right)^2-\frac{1}{3}\right]^{1/2} \tag{A2}$$

The number of face struts per unit area of the platform (assuming an equal number of struts in each face) is:

$$n_f/A=4\sqrt{3}/l_f^2 \tag{A3}$$

The number of core struts per unit area of the platform is:

$$n_c/A=2\sqrt{3}/l_f^2 \tag{A4}$$

The number of cluster (nodal) joints per unit area of the platform is:

$$n_j/A=4/\sqrt{3}\ l_f^2 \tag{A5}$$

The platform bending stiffness is given as:

$$D_T=\frac{3\sqrt{3}}{8}E_fA_fl_f\left[\left(\frac{l_c}{l_f}\right)^2-\frac{1}{3}\right] \tag{A6}$$

which assumes that the platform may be idealized as a sandwich plate with isotropic face sheets and a rigid core.

The platform fundamental frequency may be expressed in terms of Eq. (A6) as:

$$f_T=\frac{25.93}{2\pi D^2}\left[\frac{D_Tg_\xi}{(M/A)_{\mathrm{sys}}}\right]^{1/2} \tag{A7}$$

where $(M/A)_{\mathrm{sys}}$ is the total system mass per unit area and includes strut mass, joint mass, and payload mass. All system mass is assumed to be uniformly distributed. The constant g_ξ is defined as:

$$g_\xi=1\frac{\mathrm{kg\cdot m}}{\mathrm{N\cdot s^2}}=32.2\frac{\mathrm{lbm\cdot ft}}{\mathrm{lbf\cdot s^2}} \tag{A8}$$

The constant, 25.93, was determined from a finite element vibrational analysis of a freely supported hexagonal plate.

The face strut load due to strut length error is given by:

$$P_l=(\delta/l)E_fA_f \tag{A9}$$

For the core strut, A_f is replaced by A_c.

Deployable Truss Equations

The face strut fundamental frequency, assuming simply supported ends, may be approximated as:

$$f_s=\frac{\pi}{2l_f^2}\left\{\frac{E_fI_fg_\xi(1-P/P_E)}{\rho_fA_f+[\pi m_j/(48l_f)]}\right\}^{1/2} \tag{A10}$$

where m_j is the appropriate center joint mass, ρ_f is the strut mass density, and I_f is the strut moment of inertia. For a core strut the subscript f is replaced by c.

The fact strut extensional stiffness for initially curved struts can be approximated as:

$$(E_fA_f)_{\mathrm{curved}}=(E_fA_f)_{\mathrm{perf}}\left[\frac{1}{1+8/15(\delta/r_g)^2/(1-P/P_E)^3}\right] \tag{A11}$$

where r_g is the strut radius of gyration and P is the strut load.

Erectable Truss Equations

The strut fundamental frequency for a nestable strut, assuming simply supported ends, may be approximated as:

$$f_{s(\mathrm{nest})}=kf_{s(\mathrm{cyl})} \tag{A12}$$

where $k=f(d_1/d_2)=1.08$ for $0.4\leq d_1/d_2\leq 0.5$. The term $f_{s(\mathrm{cyl})}$ is computed from Eq. (A10) using average values for the nestable strut cross-sectional area and moment of inertia.

The strut Euler buckling load may be expressed as:

$$P_E=mEI_2/l^2 \tag{A13}$$

where I_2 is the maximum moment of inertia of the strut cross section. The factor m is given as a function of the minimum-to-maximum diameter ratio of the strut.[7]

The face strut extensional stiffness can be expressed as a function of the minimum-to-maximum diameters of the strut.[7] The result is:

$$\frac{(E_f A_f)_{\text{tapered}}}{\overline{E_f A_f}} = \frac{2(1-\lambda)}{(1+\lambda)\ln(1/\lambda)} \tag{A14}$$

where $\overline{E_f A_f}$ is the extensional stiffness of an equal mass cylinder, and $\lambda = d_1/d_2$.

Gravity Gradient Considerations

The internal strut loads are assumed to be induced by a distributed line moment which acts about an axis that passes through the center of the truss.[5] This line moment counteracts gravity gradient forces to hold the truss at a given inclination. The moments of inertia required to compute this line moment for the hexagonal planform truss structure shown in Fig. 1 may be approximated as:

$$I_x = I_y = \frac{M_f}{48}(5D^2 + 24H^2) + \frac{M_c}{96}(5D^2 + 8H^2)$$

$$I_z = \frac{5}{48}(2M_f + M_c)D^2 \tag{A15}$$

where M_c is the total mass of the core struts and M_f is the mass of the network of struts in one surface of the platform.

The gravity gradient load in the face struts is most critical when the 0-deg struts are aligned in the direction of loading[5] and may be expressed as:

$$P_{gg} = \frac{15\sqrt{3}}{256}\frac{g_0}{g_\xi}\frac{R_e^2}{R^3}\frac{(M/A)_{\text{sys}}D^3}{[(l_c/l_f)^2 - (1/3)]^{1/2}}\left[1 - \frac{1}{2}\left(\frac{M_c}{M_{\text{sys}}} + \frac{2M_f}{M_{\text{sys}}}\right)\right] \tag{A16}$$

Orbital Transfer Considerations

The maximum acceleration of a platform occurs at the end of the apogee burn when the resultant mass is at a minimum under a constant thrust. With the assumption of thrust load being reacted by the nine core struts associated with the three centermost points on the back surface of a hexagonal platform, the maximum core strut load can be obtained from equilibrium considerations in the direction of the thrust. Negligible loads occur in the three centermost core struts. The remaining six core struts then equally react the thrust as follows:

$$P_{ot} = \frac{1}{6}\frac{l_c}{(l_c^2 - \frac{1}{3}l_f^2)^{1/2}}(T - M_{\text{dry}}a_{\text{end}}) \tag{A17}$$

where a_{end} is the acceleration at the end of the apogee burn, and M_{dry} is the dry mass or the mass other than the platform structure and payload that remains at the end of the apogee burn. Small local inertia loads have been neglected. Equation (A17) may be rewritten as:

$$P_{ot} = \frac{1}{6}\frac{l_c}{(l_c^2 - \frac{1}{3}l_f^2)^{1/2}}\frac{T}{W_0}\frac{M_0}{M_{\text{sys}}}M_{\text{sys}}g_0\left(\frac{1}{1 + M_{\text{dry}}/M_{\text{sys}}}\right) \tag{A18}$$

where W_0 is the startburn weight referenced to the Earth's surface. The ratio of startburn mass M_0 to system mass and the ratio of dry mass to system mass may be obtained as a function of T/W_0 from interpolation of data produced by the system of computer programs AVID[12,13] for aerospace vehicle interactive design.

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